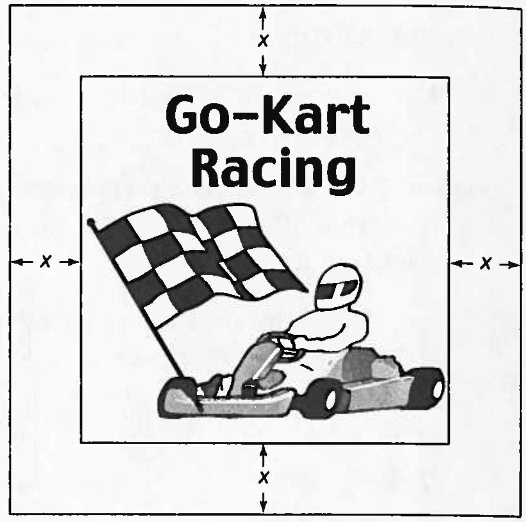
# 5.6 Factor a Perfect Square Trinomial and a Difference of Squares

One of the sponsors for the school yearbook has asked that the area of the art in their advertisement be increased by the same amount on all sides. The expression for the area of the enlarged art is given by $4x^2 + 12x + 9$, which is a perfect square trinomial.




In Section 5.2, you learned that some polynomial products can be expanded using special patterns. Similarly, you can factor polynomials that are perfect square trinomials or differences of squares using special patterns.

## Investigate A

### How can you use patterns to factor a difference of squares?

**Method 1: Use Pencil and Paper**

1. Expand and simplify or use the pattern for the product of the sum and the difference of two terms from Section 5.2.

   a) $(x + 1)(x - 1)$  b) $(y + 2)(y - 2)$

   c) $(3c - 10)(3c + 10)$  d) $(2m - 4)(2m + 4)$

2. How are the two binomials being multiplied in step 1 alike? How are they different?

3. Consider each simplified expansion from step 1.

   a) How is the first term related to the first terms of the two binomials?

   b) How is the last term related to the last terms of the two binomials?

4. Reflect Each resulting product in step 1 is a difference of squares. Explain how you can identify a difference of squares.

5. Confirm that each polynomial is a difference of squares. Then, use the reverse process to factor each. Check by expanding and simplifying.

   **a)** $x^2 - 25$ **b)** $y^2 - 36$

   **c)** $16k^2 - 49$ **d)** $25n^2 - 144$

6. **Reflect** Write a rule for factoring a difference of squares.

7. Use your rule to factor $100y^2 - 49x^2$. Check by expanding and simplifying.

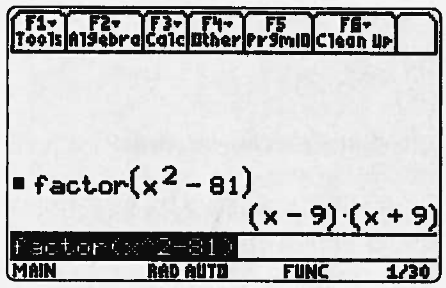
**Method 2: Use a Computer Algebra System (CAS)**

**Tools**

- TI-89 calculator

1. Clear the calculator's memory by selecting **2:NewProb** from the **Clean Up** menu.

2. Use the **Factor** function on each polynomial. Record the results.




   **a)** $x^2 - 81$

   **b)** $y^2 - 64$

   **c)** $25d^2 - 36$

   **d)** $16k^2 - 121$

   **e)** $144b^2 - 25k^2$

   **f)** $4n^2 - 49p^2$

3. **Reflect** Each polynomial in step 2 is a difference of squares. Explain how you can identify a difference of squares.

4. Consider each pair of binomial factors from step 2.

   **a)** How are they alike? How are they different?

   **b)** How are the first terms of the factors related to the first term of the polynomial?

   **c)** How are the last terms of the factors related to the last term of the polynomial?

5. **Reflect** Write a rule for factoring a difference of squares.

## Investigate B

**How can you use patterns to factor a perfect square trinomial?**

**Method 1: Use Pencil and Paper**

1. Expand and simplify or use the pattern for squaring a binomial from Section 5.2.

   **a)** $(x + 3)^2$ **b)** $(y - 5)^2$ **c)** $(k + 7)^2$

   **d)** $(2h + 3)^2$ **e)** $(3b - 5)^2$

2. Consider each simplified expansion from step 1.
   a) How is the first term in each trinomial related to the first term in each binomial?
   b) How is the last term in each trinomial related to the last term in each binomial?
   c) How is the middle term in each trinomial related to the terms in the binomial?

3. Reflect Each resulting product in step 1 is a perfect square trinomial. Explain how you can identify a perfect square trinomial.

4. Confirm that each polynomial is a perfect square trinomial. Then, use the reverse process to factor each. Check by expanding and simplifying.
   a) $x^2 + 12x + 36$
   b) $y^2 - 6y + 9$
   c) $4k^2 + 20k + 25$
   d) $9k^2 - 24k + 16$

5. Reflect Write a rule for factoring perfect square trinomials.

6. Test your rule by factoring $x^2 + 14x + 49$. Check by expanding and simplifying.

**Tools**
- TI-89 calculator

**Method 2: Use a CAS**

1. Clear the calculator's memory by selecting **2:NewProb** from the **Clean Up** menu.

2. Use the **Factor** function on each polynomial. Record the results.
   a) $x^2 + 8x + 16$
   b) $y^2 - 10y + 25$
   c) $4k^2 - 20k + 25$
   d) $9k^2 + 24k + 16$
   e) $25t^2 + 30t + 9$
   f) $16z^2 - 8z + 1$

3. Consider each resulting square of a binomial from step 2.
   a) How is the first term of the binomial related to the first term of the trinomial?
   b) How is the last term of the binomial related to the last term of the trinomial?
   c) How are the terms of the binomial related to the middle term of the trinomial?

4. Reflect Each polynomial in step 2 is a perfect square trinomial. Explain how you can identify a perfect square trinomial.

5. Reflect Write a rule for factoring perfect square trinomials.

In Section 5.2, you saw that $(a + b)(a - b) = a^2 - b^2$. You can factor a difference of squares as $a^2 - b^2 = (a + b)(a - b)$.

You also saw that $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$. You can factor a perfect square trinomial as $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$.

## Example 1 Difference of Squares

Factor.

**a)** $x^2 - 100$

**b)** $98a^2 - 450b^2$

### Solution

**a)**
$$\begin{aligned} a^2 - b^2 &= (a + b)(a - b) \\ x^2 - 100 &= (x)^2 - 10^2 \\ &= (x + 10)(x - 10) \end{aligned}$$

Use the pattern for a difference of squares.

**b)**
$$\begin{aligned} 98a^2 - 450b^2 &= 2(49a^2 - 225b^2) \\ &= 2[(7a)^2 - (15b)^2] \\ &= 2(7a + 15b)(7a - 15b) \end{aligned}$$

Remove the greatest common factor.

Factor the difference of squares.

## Example 2 Perfect Square Trinomials

Verify that each trinomial is a perfect square. Then, factor.

**a)** $x^2 + 6x + 9$

**b)** $x^2 - 12x + 36$

### Solution

**a)** Since $x^2 = (x)^2$ and $9 = 3^2$, the first and last terms are perfect squares.
Since $6x = 2(x)(3)$, the middle term is twice the product of the square roots of the first and last terms.
Therefore, $x^2 + 6x + 9$ is a perfect square trinomial.

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)^2 \\ x^2 + 6x + 9 &= (x)^2 + 2(x)(3) + 3^2 \\ &= (x + 3)^2 \end{aligned}$$

Use the appropriate perfect square trinomial pattern.

**b)** Since $x^2 = (x)^2$ and $36 = 6^2$, the first and last terms are perfect squares.
Twice the product of these square roots is $2(x)(6) = 12x$.
Therefore, $x^2 - 12x + 36$ is a perfect square trinomial.

$$\begin{aligned} a^2 - 2ab + b^2 &= (a - b)^2 \\ x^2 - 12x + 36 &= (x)^2 - 2(x)(6) + 6^2 \\ &= (x - 6)^2 \end{aligned}$$

Use the appropriate perfect square trinomial pattern.

The middle term of the trinomial is $-12x$, so a difference has been squared.

## Example 3 More Complex Perfect Square Trinomials

Verify that each trinomial is a perfect square. Then, factor.

**a)** $4x^2 + 28x + 49$

**b)** $25k^2 - 60km + 36m^2$

### Solution

**a)** Since $4x^2 = (2x)^2$ and $49 = 7^2$, the first and last terms are perfect squares.
Since $28x = 2(2x)(7)$, the middle term is twice the product of the square roots of the first and last terms.
Therefore, $4x^2 + 28x + 49$ is a perfect square trinomial.

$$\begin{aligned} 4x^2 + 28x + 49 &= (2x)^2 + 2(2x)(7) + 7^2 \\ &= (2x + 7)^2 \end{aligned}$$

The middle term of the trinomial is $-60km$, so a difference has been squared.

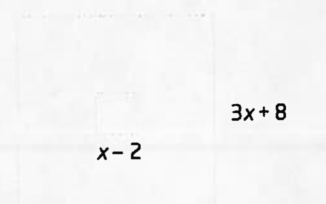
**b)** Since $25k^2 = (5k)^2$ and $36m^2 = (6m)^2$, the first and last terms are perfect squares.
Twice the product of these square roots is $2(5k)(6m) = 60km$.
Therefore, $25k^2 - 60km + 36m^2$ is a perfect square trinomial.

$$\begin{aligned} 25k^2 - 60km + 36m^2 &= (5k)^2 - 2(5k)(6m) + (6m)^2 \\ &= (5k - 6m)^2 \end{aligned}$$

## Example 4 Area of a Region

**a)** Find an algebraic expression for the area of the shaded region.

**b)** Write the area expression in factored form.




### Solution

**a)** The area of the shaded region is the difference in the areas of the two squares.
Area $= (3x + 8)^2 - (x - 2)^2$

**b)** **Method 1: Expand, Then Factor**

$$\begin{aligned} &(3x + 8)^2 - (x - 2)^2 \\ &= 9x^2 + 48x + 64 - (x^2 - 4x + 4) \\ &= 9x^2 + 48x + 64 - x^2 + 4x - 4 \\ &= 8x^2 + 52x + 60 \\ &= 4(2x^2 + 13x + 15) \\ &= 4(2x^2 + 10x + 3x + 15) \\ &= 4[(2x^2 + 10x) + (3x + 15)] \\ &= 4[2x(x + 5) + 3(x + 5)] \\ &= 4[(x + 5)(2x + 3)] \\ &= 4(x + 5)(2x + 3) \end{aligned}$$